Problem 1.5 Alternate Solutions

$$n\langle x\rangle = \frac{n a e^{-Fa/kT}}{1+e^{-Fa/kT}} \approx \frac{na(1-Fa/kT)}{2-Fa/kT}$$

$$= \frac{na}{2-\cancel{Fa/kT}^{\,0}} - \frac{nFa^2}{kT}\frac{1}{2-\cancel{Fa/kT}^{\,0}}$$

$$= \frac{na}{2} - \frac{nFa^2}{2kT}$$

$$n\langle x\rangle = \frac{na}{1+e^{Fa/kT}} \approx \frac{na}{2+Fa/kT}$$

How do we write this as $c_0 + c_1 F$, saving terms on the order of $\frac{Fa}{kT}$ and ignoring terms on the order $\left(\frac{Fa}{kT}\right)^2$?

$$\frac{na}{2+Fa/kT} \rightarrow \frac{na}{2+\theta} \text{ as } \theta \to 0 \text{ and Taylor Expand}$$

$$= \left.\frac{na}{2+\theta}\right|_{\theta=0} + \frac{d}{d\theta}\left[\frac{na}{2+\theta}\right]_{\theta=0}\theta + \frac{1}{2}\frac{d^2}{d\theta^2}\left[\frac{na}{2+\theta}\right]_{\theta=0}\theta^2 + \ldots$$

$$= \frac{na}{2} - \left.\frac{na}{(2+\theta)^2}\right|_{\theta=0}\theta + \cancel{O(\theta^2)}$$

$$= \frac{na}{2} - \frac{nFa^2}{4kT} \approx n\langle x\rangle$$

Why not Taylor Expand from the beginning?

$$n\langle x\rangle = \frac{na}{1+e^{Fa/kT}} \rightarrow \frac{na}{1+e^{\theta}} \text{ as } \theta \to 0$$

$$= \left.\frac{na}{1+e^{\theta}}\right|_{\theta=0} + \frac{d}{d\theta}\left[\frac{na}{1+e^{\theta}}\right]_{\theta=0}\theta + \frac{1}{2}\frac{d^2}{d\theta^2}\left[\frac{na}{1+e^{\theta}}\right]_{\theta=0}\theta^2 + \ldots$$

$$= \frac{na}{2} + \left[\frac{-na e^{\theta}}{(1+e^{\theta})^2}\right]_{\theta=0}\theta + \cancel{O(\theta)^2}$$

$$n\langle x\rangle \approx \frac{na}{2} - \frac{nFa^2}{4kT}$$

I would accept either answer